RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER TAKE-HOME TEST, SEPTEMBER 2020
FIRST YEAR [BATCH 2019-22]

Date : 28/09/2020 **MATHEMATICS GENERAL**
Time : 11.00 am – 3.00 pm **Paper** : II Full Marks: 50

Instructions to the Candidates

- Write your **College Roll No, Year, Subject** & **Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject** & **Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be admissible.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer script must be numbered serially by hand.
- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the test, you should scan the entire answer script by using Clear Scan: Indy Mobile App/any other Scanner device and make a **single file of PDF** format in your own name and send or share it to [redacted]

Group: A
Ordinary Differential Equation

Answer any **four** questions from question no. 1-6 in this group. [4 x 5 = 20 marks]

1. Find the particular solution of the differential equation :$(\cos y)dx + (1 + 2e^{-x})(\sin y)dy = 0$; $y = \frac{\pi}{4}$ when $x = 0$.
2. Prove that the differential equation of all circles touching x-axis at the origin is
$$(x^2 - y^2)dy - 2xydx = 0.$$
3. Solve: $\frac{dy}{dx} = \frac{y(y+x)}{x(y-x)}$.
4. Solve: $xdx + ydy + \frac{xdy-ydx}{x^2+y^2} = 0$
5. Solve: $\frac{dy}{dx} + \frac{y}{x} = x^2$, given $y = 1$ when $x = 1$.

6. Solve: $\frac{dy}{dx} + \frac{\sin(2y)}{x} = x^3(\cos y)^2$.

Group: B

Calculus

Answer any **six** questions from question no. 7-16. [6 x 5 = 30 marks]

7. a. Let $\{x_n\}$ be a sequences of real numbers where $x_n = -\frac{n}{n+1}$. State whether $\{x_n\}$ is bounded or not. If possible find $\sup\{x_n\}$ and $\inf\{x_n\}$. [3]
   b. State whether the sequence $\left\{\frac{n+1}{3n+1}\right\}$ is monotone increasing or monotone decreasing. [2]

8. Prove that the sequence $\{u_n\}$ defined by $u_1 = \sqrt{5}$ and $u_{n+1} = \sqrt{5u_n}$ for $n \geq 1$ is convergent. Also find $\lim_{n\to\infty} u_n$. [4+1]

9. a. Let $\lim_{n\to\infty} a_n = 0$. Does it always follows the series $\sum_{n=1}^{\infty} a_n$ is convergent? Explain it with the help of an example. [3]
   b. Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be two series of real numbers such that $\sum_{n=1}^{\infty} a_n b_n$ is convergent. Does it always implies $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent ? [2]

10. Test the convergence of the following series :
    a. $\sum_{n=1}^{\infty} \frac{4n+3}{4n(n+1)(n+3)}$ [3]
    b. $1 + \frac{1}{2^3} + \frac{1}{2^2} + \frac{1}{2^5} + \frac{1}{2^4} + ...$ [2]

11. a. Test the convergence of the series $\sum_{n=1}^{\infty} e^{-n^3}$ [2]
    b. Sketch the graph of the function $f(x) = \sin x,\ x \in \mathbb{R}$. Hence determine the continuity of f on $\mathbb{R}$. [3]

12. a. Prove that $\frac{5}{x-1} + \frac{7}{x-2} + \frac{16}{x-3} = 0$ has one solution between 1 and 2 and another solution between 2 and 3. [3]
    b. Let
    $$f(x) = \begin{cases} \frac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} & , x \neq 0 \\ 0 & , x = 0. \end{cases}$$
    Check whether f is continuous at x=0 . [2]

13. A function is defined in $(0, \infty)$ by
    $$f(x) = \begin{cases} 1 - x^2 & , 0 < x \leq 1 \\ \log x & , 1 < x \leq 2 \\ \log 2 - 1 + \frac{1}{2}x & , 2 < x < \infty. \end{cases}$$
    Obtain the derived function f' and its domain. [5]

14. a. Verify Rolle's theorem for $f(x) = x^3 - 6x^2 + 11x - 6,\ x \in \mathbb{R}$. [3]
    b. If a_1, a_2, a_3 be all positive numbers. Find $\lim_{n\to\infty} \left\{x - \sqrt[3]{(x-a_1)(x-a_2)(x-a_3)}\right\}$. [2]

15. If $y = \cos(m \sin^{-1} x)$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$. [5]

16. a. Find the power series expansion of the function $f(x) = \cos x,\ x \in \mathbb{R}$. [3]

b. Let f be a real valued function defined over $[-1, 1]$ such that

$$f(x) = \begin{cases} x \cos \frac{1}{x} & , x \neq 0 \\ 0 & , x = 0. \end{cases}$$

Does the Lagrange's mean value theorem holds for f in $[-1, 1]$. [2]

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